

Second Term Evaluation - 2025

இரண்டாம் துவணைப் பரீட்சை - 2025

ශ්‍රේණිය Grade	13	විෂයය Subject	Combined Mathematics I	කාලය Time	3h 10 min
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❖ This paper consists of two parts.
Part A (Questions 1-10) & Part B (Questions 11-16)

Answer **all** the questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.

Answer **five** questions only. Write your answers on the sheets provided.

- ❖ At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.

You are permitted to remove **only Part B** of the question paper from the examination hall.

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(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
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B	11	
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In words	
In numbers	

Code Numbers

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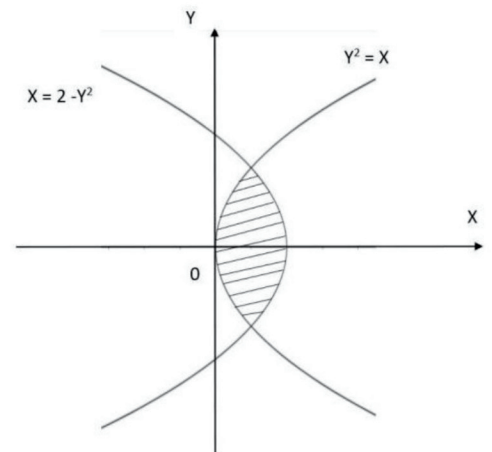
3. Using the ten digits from 0 to 9, it is necessary to form 6 digits secret codes of which the first two digits are 2 or 8 and without repeating any digit. How many such secret codes can be formed? How many of them are even numbers?

4. Find the real value of a such that $\lim_{x \rightarrow 0} \frac{[(a^3 + x)^{\frac{1}{3}} - a] \sin 2x}{x^2} = \frac{1}{6}$. Where $a > 0$

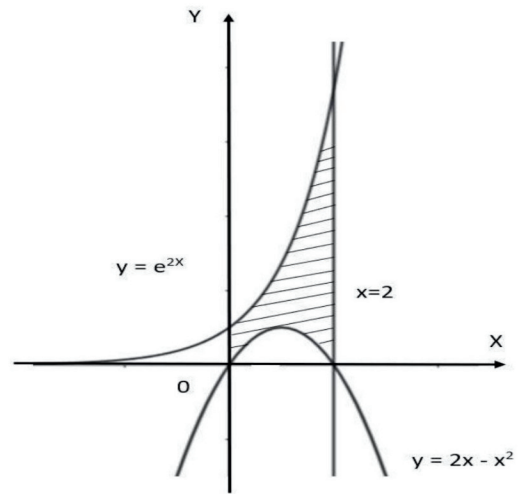
5. The parametric form of a curve is given by $x = \sec^2 t$ and $y = \cos t$, where t is a parameter.

Show that the equation of the tangent drawn to the curve at the point P corresponding to $t = \frac{\pi}{4}$, is given by $x + 4\sqrt{2}y - 6 = 0$

6. Show that the area of the shaded region enclosed by the two symmetric graphs $y^2 = x$ and $x = 2 - y^2$ is $\frac{8}{3}$ sq. units.



7. The shaded area is enclosed by the two curves, $y = e^{2x}$, $y = 2x$ and the straight lines $x = 0$ and $x = 2$. Show that the volume of the solid generate by rotating the shaded area about the x – axis in 2π radius is, $\frac{\pi}{60}(15e^8 - 79)$ cubic units.



8. The equation of a side of a square is given by $x - 2y = 0$. Both diagonals meet at the point $\left(\frac{5}{2}, \frac{5}{2}\right)$. Find the equations of both diagonals.

10. If $\alpha + \theta = \frac{\pi}{6}$, then show that $(\sqrt{3} + \tan \theta)(\sqrt{3} + \tan \alpha) = 4$. Deduce the value of $\tan \frac{\pi}{12}$.

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Combined Mathematics I

11. a). Let $f(x) \equiv x^2 + ax + b$ and $g(x) = x^2 + acx + bc^2$ for $a, b, c \in \mathbb{R}$ and $c \neq 1$.
If α and β are real roots of the quadratic equation $f(x) = 0$, write $\alpha + \beta$ and $\alpha\beta$, in terms of a and b .

If one root is square of other root of $f(x) = 0$, deduce that $a^3 + b^2 + b = 3ab$.

If both $f(x) = 0$ and $g(x) = 0$, hold a common root, then show that $b(c + 1)^2 = ca^2$.

If $(x - 2)$ is a factor of $f(x)$, find the value of p .

For this value of p , express $f(x)$ in the form of $f(x) \equiv (x^2 + x + a)(x^2 + x + b)$., where a and b are constants to be determine.

Hence express $f(x)$ as a product of linear factors.

Solve the inequality $f(x) \geq 0$.

Deduce the remainder when the function $x^4 + 2x^3 - 25x^2 - 26x$ is divided by $(x^2 + 3x - 10)$.

Find a function $f(r)$ such that $u_r = 1 + f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.

Hence show that $\sum_{r=1}^n u_r = n+1 - \frac{1}{2n+1}$.

Deduce that the series $\sum_{r=1}^{\infty} u_r$ is not convergent.

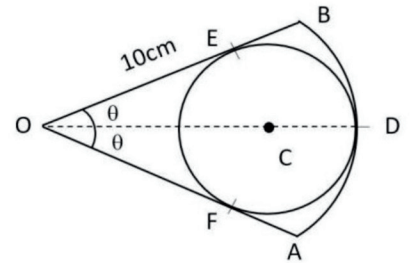
Is the series $\sum_{r=1}^{\infty} (u_r - 1)$ convergent? Justify your answer.

b). A team of four students should be select for a competition, from a group of students consisting 3 male students and 2 female students from the Bio stream, 4 male students and 2 female students from Maths stream.

Find the number of different ways in which the team can be formed in each of the following cases.

- If 2 male students and 2 female students in the team.
- At least 2 male students from the Maths stream and a female student from the Bio stream.

13. a). The figure shows a sector OAB of a circle of center O and radius 10 cm. $\widehat{AOB} = 2\theta$ radians when $0 < \theta < \frac{\pi}{2}$. This circle of Center C and radius r touches the AB at D. OA and OB are two tangents to this circle at E and F respectively.



Write down OC in terms of r and show that $r = \frac{10 \sin \theta}{1 + \sin \theta}$, where θ is a variable.

Find $\frac{dr}{d\theta}$ for $r = \frac{10}{3}$.

If r is increasing at a rate of 2 cms^{-1} , find the rate at which θ is increasing when $\theta = \frac{\pi}{6}$.

- b). Let $f(x) = \frac{ax+b}{(x+a)^2}$ for $x \neq (-a)$ and $a, b \in \mathbb{R}$.

It is given that the vertical asymptotes of the graph is $x = (-3)$. Also, the graph cuts the x-axis at $x = -\frac{2}{3}$. Find the values of a and b .

Show that $f'(x) = \frac{5-3x}{(x+a)^3}$ for this value of a . where $x \neq (-a)$

Find the interval on which the function $f(x)$ is increasing and the intervals on which the function $f(x)$ is and decreasing.

Let $f''(x) = \frac{3(2x-8)}{(x+a)^4}$. Sketch the graph of $y = f(x)$ indicating turning points, asymptotes, axis intercepts and points of inflection.

Hence state the range of $f(x)$.

Find the value of k , such that $k \in \mathbb{R}^+$ of which the function $f(x)$ is one to one, on the interval $x \in [k, \infty)$.

14. a). Find the values A, B , and C such that $1 \equiv A(x^2 + 8) + (Bx + C)(x + 2)$; where $A, B, C \in \mathbb{R}$.

Hence evaluate $\int \frac{24}{(x+2)(x^2+8)} dx$

- b). Using integration by parts evaluate $\int_0^1 x^2 e^{-x} dx$.

Hence find $\int_0^1 x^5 e^{-x^2} dx$, using a suitable substitution.

Find the real values of λ and μ such that the area bounded by the curve $y = 2x^5 e^{-x^2}$ and the lines $x = 1, x = 0$ and $y = 0$ is $\lambda e^{-1} + \mu$.

- c). Let $I_n = \int \frac{x^n}{\sqrt{x^2+5}} dx$

Show that $nI_n + 5(n-1)I_{n-2} = x^{n-1}\sqrt{5+x^2}$, for $n \geq 2$

Hence show that $\int_0^2 \frac{x^5}{\sqrt{x^2+5}} dx = \frac{168}{5} - \frac{40\sqrt{5}}{3}$

15. Prove that the perpendicular distance drawn to a line $px + qy + r = 0$ from a fixed point

$$A(\alpha, \beta) \text{ is given by } \frac{|p\alpha + q\beta + r|}{\sqrt{p^2 + q^2}}, \text{ when } p, q, r, \alpha, \beta \in \mathbb{R}$$

The perpendicular distance from the point $P(a, b)$ to each of the lines $y = 3x$ and $x = 3y$ is equal. Find a in terms of b , where $a, b \in \mathbb{R}$.

Given that a and b are both positive, deduce that the perpendicular distance to any of the above

$$\text{two lines from } P(a, b) \text{ is } \frac{a\sqrt{10}}{5}.$$

Hence show that there exist two circles pass through the point $Q(2, 1)$ and touching both lines $y = 3x$ and $x = 3y$.

Also show that the equation of one of the circles is given by $2x^2 + 2y^2 - 5x - 5y + 5 = 0$ and find the equation of the other circle.

Show also that these two circles intersect each other at two distinct points.

Find the equation of the common chord.

16. a) For $0 < \theta < 2\pi$, it is given that $\sin \theta + \cos \theta = \sin 2\theta + \cos 2\theta$. Show that $\tan \frac{3\theta}{2} = 1$.

Hence find all solutions of the equation $\sin 2\theta - \sin \theta + \cos 2\theta - \cos \theta = 0$.

- b). State the Cosine rule for a triangle ABC , from the usual notation.

If a, b and c are successive terms of an arithmetic progression,

show that $\cos A + \cos C + 2\cos B = 2$

$$\text{Deduce that } \cos\left(\frac{A-C}{2}\right) = 2\sin\frac{B}{2}$$

- c). Let $f(x) \equiv \cos 2x - \sqrt{3}[\cos x + \sin x]^2$

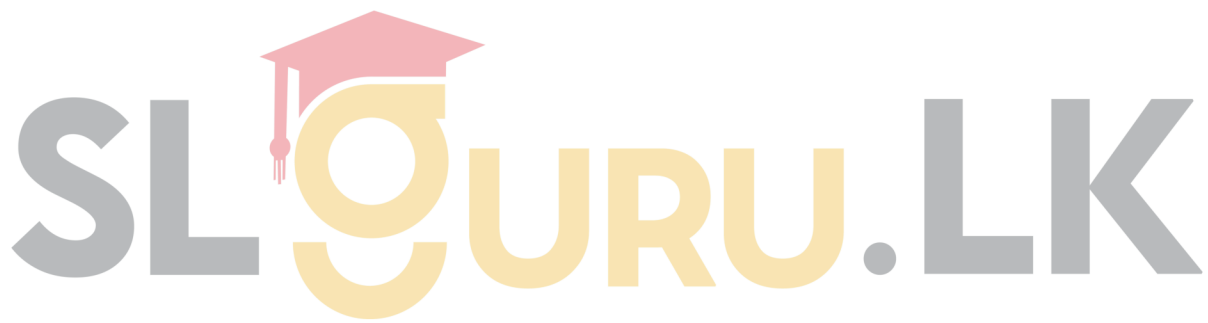
Find λ, μ and α when $\lambda, \mu \in \mathbb{R}$ and $0 < \alpha < \frac{\pi}{2}$ such that $f(x) \equiv \lambda \cos(2x + \alpha) + \mu$

Hence draw the graph of $y = f(x)$ in the range $-\pi/4 \leq x \leq 3\pi/4$.

- d). Show that $2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{12}{5} = \frac{\pi}{2}$

$$\text{Deduce that } \sin\left\{\frac{\pi}{4} - \frac{1}{2} \tan^{-1} \frac{12}{5}\right\} = \frac{1}{\sqrt{26}}$$

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ශ්‍රේණිය 13
 Grade

විෂයය
Subject

Combined Mathematics II

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Time

Instructions:

- You are permitted to remove **only Part B** of the question paper from the examination hall.

(10) Combined Mathematics II

Total

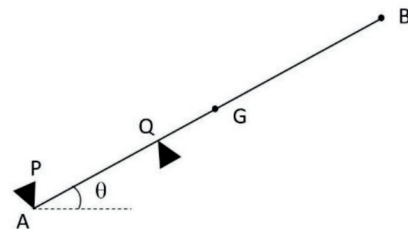
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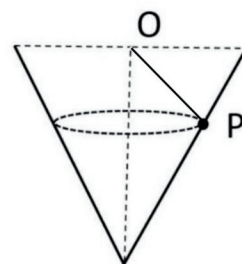
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- Glossary**

4. AB is a uniform rod of length $2a$. G is the center of gravity of AB. Its end A is in contact with a smooth peg P, and fall over another rough peg Q as shown in the figure. The rod is in limiting equilibrium, inclining θ to horizontal, and the coefficient of friction between the rod and the peg Q is $\frac{1}{4} \tan \theta$. Show that the length PQ is $\frac{a}{4}$.



5. A hollow cone of semi vertical angle α is fixed vertically, with its vertex downward as shown in the figure. A particle P of mass m has placed on the smooth inner surface of the cone and connected to the center O, by an inelastic string of length a . This string makes an angle α with the downward vertical. When this particle is subjected to an angular velocity ω , show that the tension of the string is



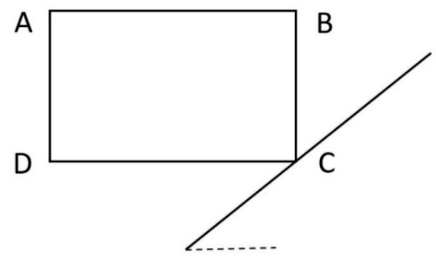
$$T = \frac{m[g \cos \alpha - a\omega^2 \sin^2 \alpha]}{\cos 2\alpha}$$

- [illegible]

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[illegible]

9. A uniform rectangular lamina ABCD of weight 64 N has side $AB = 120\text{ cm}$ and $AD = 32\text{ cm}$, is in equilibrium, in a vertical plane with corner A is freely hinged to a fixed point and the corner C is in contact with a smooth plane inclined at angle $\tan^{-1} \frac{15}{8}$ to the horizontal, and the side AB is horizontal.



Find the magnitude and direction of the hinge reaction on the lamina.

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10. Let A and B are two exhaustive events defined on the sample space Ω . Given that $P(A' \cap B) = 0.1$ and $P(B' \cap A) = 0.6$, where A' and B' denote the complement of A and B respectively. Find $P(A \cap B)$. Are the events A and B independent? Justify your answer.

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ශ්‍රේණිය Grade	විෂයය Subject	13 Combined Mathematics I I
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Part B

11. (a) A small stone P is projected vertically upward with a velocity u from a point O . When it has travelled half of it's maximum height, the stone has divided into two similar parts A and B. Instantaneously if A become rest, then show that the initial velocity of B is $\sqrt{2} u$. Draw the velocity-time graph for the motion of the particles A and B in a same diagram. Using the graph, show that the velocities of A and B are equal, when the particle A reaches the point of projection O. Also show that the vertical displacement of the particle B, from the point of projection is $\frac{u^2}{g}$, at this moment.

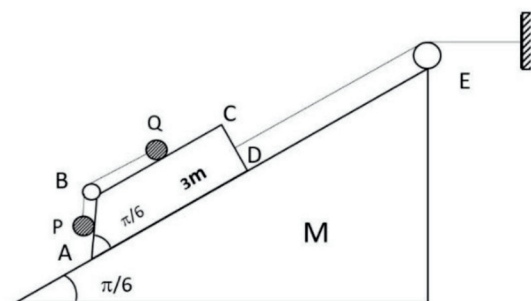
- (b) Two places A and B are at the same height above the ground level , which are at a horizontal distance a km. X and Y are two aircrafts placed at A and B respectively. Both aircrafts can fly with a constant velocity λu kmh $^{-1}$ ($\lambda > 1$) in still air. When a wind blows uniformly with a speed of u kmh $^{-1}$ from a direction making an angle $\frac{\pi}{3}$ with the line of AB ,the aircrafts X and Y start to fly simultaneously so as to complete the path from A to B and B to A ,respectively . Draw velocity triangles for the motion of both aircrafts X and Y in a same diagram. Hence show that the aircrafts will collid after a time $\frac{a}{u\sqrt{4\lambda^2-3}}$ hours.

12. (a) ABCD is the vertical cross section through the center of gravity of a smooth uniform block of mass $3m$. The face containing AD is placed on a smooth face inclined $\frac{\pi}{6}$ to horizontal, of a moveable wedge of mass M as shown in the figure.

The wedge is free to move on a smooth horizontal floor. AB line is the greatest slope of the face of ABCD plane with BC and AD are parallel faces.

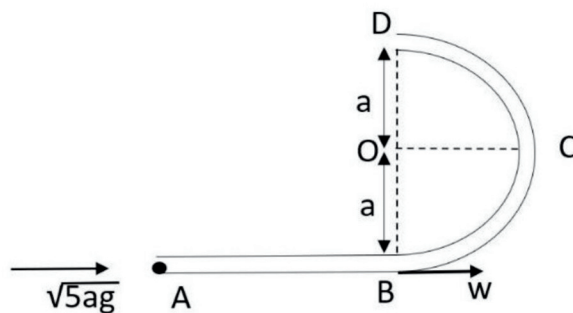
Two similar particles P and Q, each of mass m are connected by an inelastic light string, which passing over a light smooth pulley fixed to the block at B. One end of another light inextensible string connected to the line CD, passing over another light smooth pulley fixed to the block at E, and its other end attached to a vertical wall perpendicularly. Strings are taught in the position shown in the diagram and the system is released from the rest from this position.

Obtain sufficient equations to determine the acceleration of each particle, block and the wedge.



- (b) A thin tube ABCD is fixed in a vertical plane with AB horizontal as shown in the figure. BCD is a smooth semicircular part of center O and radius a , while AB is a rough thin tube of length a .

A particle of mass $2m$ is placed inside the tube at the point A, and it is given a velocity of magnitude $\sqrt{5ga}$ in the direction $\overrightarrow{AB}$. The coefficient of friction between the particle and the tube AB is $\frac{1}{2}$.



If the velocity of the particle at the point B is w , show that $w = 2\sqrt{ag}$.

Then the particle moves through the circular tube.

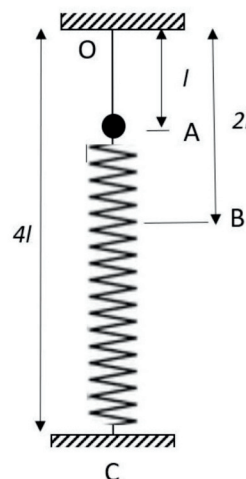
When the particle turns through an angle θ with the downward vertically, ($0 < \theta < \pi$), show that the speed v of the particle at this instant is given by $v^2 = 2ag(\cos \theta + 1)$.

If the reaction on the particle from the tube at this instant is R , show that $R = 2mg(2 + 3\cos \theta)$.

Hence find the angle of which the particle reverse the direction of its contact reaction inside the tube. Deduce that the particle could be able to just reach the highest point D.

13. One end of a light elastic string of natural length l and the modulus of elasticity mg is attached to the fixed point O and the other end is attached to a particle P of mass m .

One end of a light spring of natural length $2l$ and the modulus of elasticity mg is attached to a fixed point C, which is at distance $4l$, vertically below to O, and the other end of the string is attached to the particle P, as shown in the figure.



Now the particle P is raised to the point A, which is at a distance l from O, and gently released from rest. B is a point on OC such that $OB = 2l$.

Let $OP = x$, where $x > l$, Show that the motion of the particle from

A to B is given by $\ddot{x} + \frac{3g}{2l}(x - 2l) = 0$

Rewrite this equation in the form of $\ddot{X} + \omega^2 X = 0$ where $X = x - 2l$ and ω is a constant to be determine. Find the center of this motion.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude c and show that the velocity of the particle P, when it reaches B is $\sqrt{\frac{3gl}{2}}$.

Let D is the lowest point reached by the particle P.

Show that the motion of the particle P, from B to D is given by $\ddot{x} + \frac{g}{2l}(x - 2l) = 0$

Rewrite the equation in the form of $\ddot{Y} + \omega_0^2 Y = 0$ where $Y = x - 2l$ and ω_0 is a constant to be determined. Find the center of this motion.

Using the formula $\dot{Y}^2 = \omega_0^2(k^2 - Y^2)$, find the amplitude k , of this motion.

Show that the vertical distance from O to the lowest point D reached by the particle P is $(\sqrt{3} + 2)l$.

Also show that the total time for the motion of the particle from A to D is $\pi\sqrt{\frac{l}{6g}}(1 + \sqrt{3})$.

14. (a) Let $\underline{a}$ and $\underline{b}$ are non zero non parallel vectors and $\alpha, \beta \in \mathbb{R}$.

If $\alpha \underline{a} + \beta \underline{b} = \underline{0}$, then Show that $\alpha = 0$ and $\beta = 0$.

O, A and B are non colinear three points with position vectors $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = \underline{a} + 2\underline{b}$.

D is a point on BC such that $BD : DC = 1 : 2$.

Find the position vector $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$.

The line AB and OD meet at E. Explain why $\overrightarrow{AE} = \lambda \overrightarrow{AB}$ and $\overrightarrow{OE} = \mu \overrightarrow{OB}$ where $\lambda, \mu \in \mathbb{R}$.

Using vector triangle addition, write $\overrightarrow{OE}$ in terms of $\underline{a}$ and $\underline{b}$. Hence show that $\overrightarrow{OE} = \frac{1}{5}(\underline{a} + 4\underline{b})$.

- (b) Let ABC be an equilateral triangle of side a with AB horizontal. D, E and F are the mid points of AB, BC and AC respectively. Forces of magnitudes λP , μP , $8P$, $4\sqrt{3}P$, $6\sqrt{3}P$ and $10\sqrt{3}P$ acts along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{BF}$, $\overrightarrow{EA}$ and $\overrightarrow{CD}$ respectively, where λ and μ are real constants.

(i). Show that this system could not be in equilibrium for any values of λ and μ .

(ii). If the system is reduced to a couple, find the value of λ and μ . Also find the magnitude and sense of this couple.

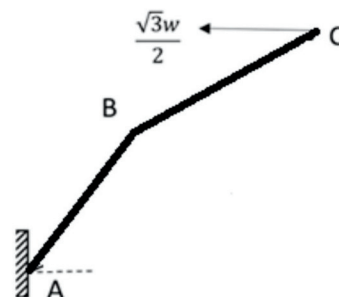
(iii). If the system is reduced to a single force of $28P$ and a clockwise moment of $2\sqrt{3}Pa$ about the vertex A, find the values of λ and μ .

Find the direction of this single force and the distance from A to the point at which the line of action of the resultant cuts the side AB.

15. (a) AB and BC are two rods, each of weight w and in equal length. They are smoothly jointed at B, and hinged to a vertical wall at A. The system is in equilibrium in a vertical plane by means of a horizontal force of $\frac{\sqrt{3}w}{2}$ applied at C, as shown in the figure.

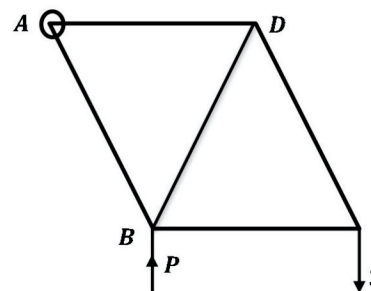
Find the acute angle made by the rod BC with horizontal and the angle subtended by the rod AB, to the upward vertical.

Also find the magnitude and direction of the reaction at the joint B, on the rod BC.



- (b) The framework shown in the adjoining figure is made from five light rods AB, BC, CD, AD and BD each of equal length freely joined at A, B, C and D. There is a load of $3w$ hanging at C.

The Framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with BC horizontal, by means of a vertical force p applied at B, as shown in the figure.



Find the value of p in terms of w .

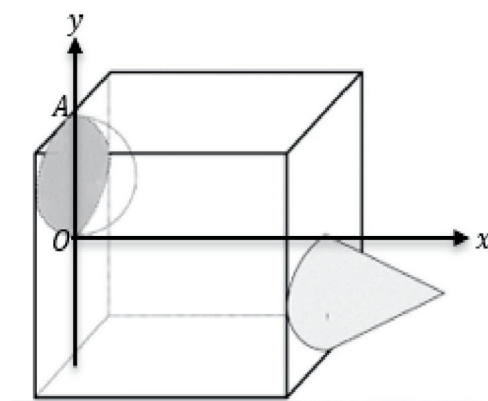
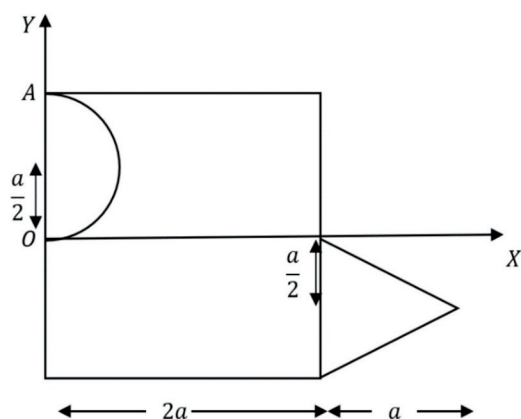
Draw a stress diagram for the joints B, C and D, using Bow's notation and hence find stress in each rod, stating whether they are tensions or trusts.

Show that the center of mass of

- (i) a uniform solid right circular cone of base radius r and height h is at a distance $\frac{h}{4}$ from the center of the base.
- (ii) a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its center.

A wooden block is in the shape of a cuboid of length $2a$ width a and height $2a$. A solid hemisphere of radius $\frac{a}{2}$, has carved out from the upper part of a side of the block.

A right circular cone of radius $\frac{a}{2}$ and height a is attached to the lower part of the opposite side of the block, as shown in the figure. The density of the cone is doubled as the density of the block.



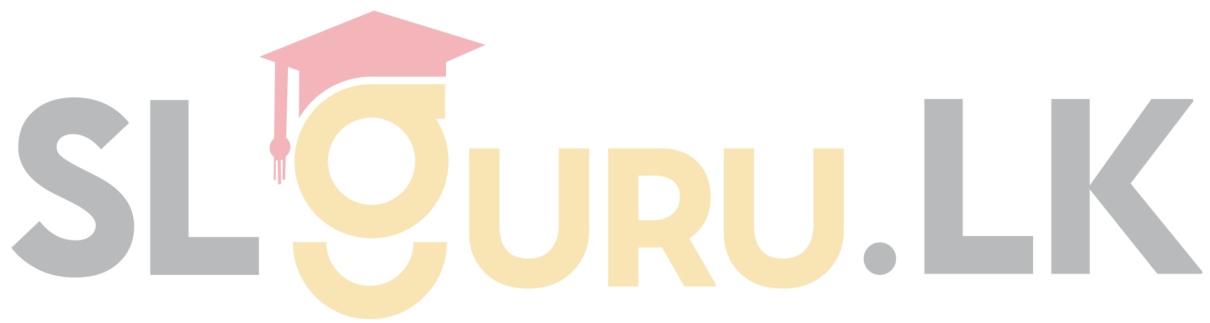
By getting the OXY axes through the midpoints through opposite sides and perpendicular to it, as shown in the figure, show that the center of gravity of the composite body lies $\frac{3a\pi}{2(48+\pi)}$ distance below OX axis and $\frac{3(256+23\pi)}{16(48+\pi)}a$ distance from OY axis.

Now the composite body is freely hinged at A . Find the inclination of OA to the vertical.

A horizontal force P is applied at the vertex of the cone such that the OX axis to be horizontal.

If the weight of the composite body is w , show that $P = \frac{(256+23\pi)}{8(48+\pi)}w$.

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